Note on Hotelling's webshop

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Több piacra épülő Hotelling webáruház

A tanulmány Lijesen modelljének [2013] egy módosított változatát mutatja be. Az eredeti modell egy webáruházzal egészíti ki a Hotelling keretrendszert. A webáruház határozza meg a rezervációs árat, és egy külső piacon is jelen van. Ehelyett azonban n-1 elkülönült piacot tételezünk fel, ahol a webáruház is értékesítheti a termékeit. A különbségek nem olyan jelentősek, mint ahogy az első pillantásra tűnhet, a kifejezések többsége megegyezik. Az árak alacsonyabbak és a hagyományos boltok lehetséges elhelyezkedései szélesebbek az új modellben. A hosszú távú modellben a piacokra újabb hagyományos boltok is beléphetnek, ha van még hely a webáruház és a két hagyományos bolt mellett. Ekkor a verseny még lejjebb hajtja az árakat.

The article shows a modification of Lijesen's model [2013] and gives insight to the comparison of the new and the previous results. The original model adds a webshop to the Hotelling framework. The webshop determines the reservation price and operates also in an external market, where the demand is linear. Instead of that n-1 separated markets, where the webshop can sell its products, are supposed in this paper. The difference is not so significant as it seems for first glance, most of the expressions are almost identical. Prices are lower and the possible locations of regular (brick and mortar) shops are wider in the new model. Considering the model as a long run process, ie. new shops can enter to markets if there is enough space for more than the two regular shops and the webshop, then the competition drives prices lower in the modified model.

1. Introduction

The growing number and increasing sales of webshops validate the actual significance of internet shopping, therefore more and more articles deal with this topic. Webshops help to make markets more transparent and force lower prices with competition. In some sense they are special, because they can be accessed faster and easier than regular shops. Nowadays most of the consumers can reach them, because internet connection is available for a significant part of the population. However, the transportation cost does not disappear, it must be paid somehow. There are different forms of the transportation cost: fixed cost in a whole country, different costs by regions, hidden in the price etc.

The field of the internet shopping is also a very complex economic phenomenon, therefore the recent published articles are very diversificated. Hu et al. [2014] demonstrates a model, where shops provide both online and offline sales and services. The consumers take into account the travel cost to the shop, delivery cost of the product and waiting time after their orders. Liu and Dukes [2015] show the role of online shopping intermediaries, a platform to connect consumers and third party sellers. The consumers decide the number of sellers to evaluate and the depth of evaluation. According to the results consumers do not take into account too many sellers. Blazewicz et al. [2014] also investigated the role of internet shopping in the case of price sensitive discounts. Consumers want to buy all required products meanwhile they want to reach the minimum total discounted price. The authors gave two computational algorithms to solve this problem. Birg [2015] deals with tax competition between cross-border countries in the aspect of webshops. The destination principle in taxation mitigates the tax competition, meanwhile the origin principle enhances it.

Lijesen [2013] introduced a webshop model, which uses a Hotelling framework with endogenous reservation price. Besides the two regular shops exist a third one, a webshop, which does not only take part in the imaginary city, but also in an external market, where the demand is expressed as a linear function. The two regular shops do not compete with each other, only with the webshop. The author states that two important contributions were made to the literature. First the spatial models with reservation price [Economidies, 1984; Böckem, 1994; Hinloopen and Marrewijk, 1999; Woeckener, 2002] were improved as the exogenous reservation price could be substituted with endogenous reservation price. Second this is the first webshop model, which uses linear city instead of Salop's circle model.

The contribution of my article that modifies the original Lijesen [2013] model. It is not clear where from the external market originates, why it can be describe with a linear demand function. If we know the exact behavior of one market, then there will be similar markets. So I suppose n Hotelling type markets with 2n regular shops and a webshop, which operates in all n markets instead of the assumption of the webshop's external demand.

First I show the original model, then my modification. After that the similarities and differences will be demonstrated between the original and the modified model. Finally, the conclusion section follows.

2. Original model

The model of Lijesen describes the imaginary city as a unit long line with two regular shops –A and B– and with a webshop –noted as 0–. Essentially A and B do not compete, because they are separated by the webshop. There is a two-stage game, first A and B determine their locations prices (x_A and x_B) in the imaginary city and after that the two shops and webshop set prices (p_A, p_B and p_0). In the case of the regular shops a transportation cost ($\tau > 0$) and in the case of the webshop a fixed delivery cost ($\theta > 0$) must be paid over the price. The consumer buys there, where he or she can buy cheaper the product. The fringes of the market are covered by the webshop, because its price is low enough for assuring that. So a Nash equilibrium exists according to Economides (1984).

From left to the right the market is owned in order: webshop, shop A, webshop, shop B and again webshop, so there are four types of marginal consumers[1]:

$$x_{0A} = \frac{1}{\tau}(tx_A + p_A - p_0 - \theta), \tag{1}$$

$$x_{A0} = \frac{1}{\tau}(p_0 + \theta - p_A + tx_A), \tag{2}$$

$$x_{0B} = \frac{1}{\tau}(tx_B + p_B - p_0 - \theta), \tag{3}$$

$$x_{B0} = \frac{1}{\tau}(p_0 + \theta - p_B + tx_B). \tag{4}$$

The first definition means the marginal consumer between the webshop and the left side of shop A. The other definitions are analogous. Moreover, there is an elastic individual demand in the alternative market of the webshop

$$\tilde{q}_0 = a - bp_0. \tag{5}$$

These assumptions result two theorems

Theorem 1 (Lijesen, 2013a). *The Nash equilibrium in prices exists if* $\theta < \frac{1}{2}\tau\left(1 - \frac{a}{bt+2}\right)$ *and is given by:* $p_0^N = \frac{1}{2b\tau+6}(\tau - 2\theta + a\tau)$, $p_A^N = p_B^N = \frac{1}{4b\tau+12}(\tau + 4\theta + a\tau + 2b\tau\theta)$.

Theorem 2 (Lijesen, 2013b). *The range of Nash equilibrium locations is given by:* $\left(\frac{1}{4\tau}\frac{\tau+4\theta+a\tau+2b\tau\theta}{b\tau+3}, \frac{1}{4\tau}\frac{9\tau-12\theta+4b\tau^2-3a\tau-6b\tau\theta}{b\tau+3}\right)$ *for firm A and* $\left(\frac{1}{4\tau}\frac{3\tau+12\theta+3a\tau+6b\tau\theta}{b\tau+3}, \frac{1}{4\tau}\frac{11\tau-4\theta+4b\tau^2-a\tau-2b\tau\theta}{b\tau+3}\right)$ *for firm B.*

3. New model

3.1. Price

There are n number different sized markets, where the webshop operates instead of the alternative market and there is two regular shops in every market, the first is *Ai* and the second one is *Bi*. The demand for the first shop in market i

$$q_{A_i} = x_{A_i0} - x_{0A_i} = \frac{2}{\tau}\left(p_0 + \theta - p_{A_i}\right). \tag{6}$$

1 For example, the first equation follows from the equality between the price of the webshop with delivery cost and price of company A with transportation cost at x_{0A} location: $p_0+\theta=p_A+\tau(x_A - x_{0A})$.

The (6) expresses that a regular shop must set lower price than webshop taking into account delivery cost to ensure positive demand. The profit and the first order condition of the price supposing zero costs are the following

$$\pi_{A_i} = p_{A_i} q_{A_i} = p_{A_i} \frac{2}{\tau}(p_0 + \theta - p_{A_i}), \tag{7}$$

$$p_{A_i} = \frac{p_0 + \theta}{2}. \tag{8}$$

The second shop in market i is symmetrical with the first one, moreover other shops in other markets are similar to these two shops too, so the demand and the prices are the same for all regular shops. Clearly the two shops in market i gain $2q_{Ai}$ from the market and the webshop owns the rest. The size of market i is δi and so we can determine the profit of the webshop in all markets.

$$\pi_0 = p_0 \left(\sum_{i=1}^{n} \left(\delta_i - 2q_{A_i} \right) \right) = p_0 \left(\sum_{i=1}^{n} \left(\delta_i - \frac{4}{\tau}(p_0 + \theta - p_{A_i}) \right) \right). \tag{9}$$

The first order condition of the profit function

$$\frac{\partial \pi_0}{\partial p_0} = \sum_{i=1}^{n} \left(\delta_i - \frac{4}{\tau}(p_0 + \theta - p_{A_i}) \right) - p_0 \sum_{i=1}^{n} \frac{4}{\tau} = 0. \tag{10}$$

Substituting (8) to calculate the optimal price of webshop

$$p_0^* = \frac{\tau \sum_{i=1}^{n} \delta_i - 2n\theta}{6n} = \frac{\tau \sum_{i=1}^{n} \delta_i}{6n} - \frac{\theta}{3}. \tag{11}$$

The (8) and (11) together give the optimal price of regular shops

$$p_{A_i}^* = p_{B_i}^* = \frac{p_0^* + \theta}{2} = \frac{\theta}{2} + \frac{\tau \sum_{i=1}^{n} \delta_i}{12n} - \frac{\theta}{6} = \frac{\tau \sum_{i=1}^{n} \delta_i}{12n} + \frac{\theta}{3}. \tag{12}$$

Finally, we can determine the optimal demand

$$q_{A_i}^* = \frac{2}{\tau} \left(\frac{\tau \sum_{i=1}^{n} \delta_i}{6n} - \frac{\theta}{3} + \theta - \frac{\tau \sum_{i=1}^{n} \delta_i}{12n} \right) - \frac{\theta}{3} = 2 \left(\frac{\theta}{3\tau} + \frac{\sum_{i=1}^{n} \delta_i}{12n} \right). \tag{13}$$

Moreover, we have to check that the demand is big enough for two companies in all markets, ie. $\delta_i - 2q^*_{Ai} > 0$. For all *i* must be

$$\frac{3\tau}{4}\left(\delta_i - \frac{\sum_{i=1}^{n}\delta_i}{3n}\right) > \theta, \tag{14}$$

or

$$\frac{3\tau}{4}\left(\min_i \delta_i - \frac{\sum_{i=1}^{n}\delta_i}{3n}\right) > \theta. \tag{15}$$

To be true this inequality, the second term must be positive

$$3\min_i \delta_i > \frac{\sum_{i=1}^{n}\delta_i}{n}. \tag{16}$$

Furthermore, the price of the webshop must be also positive, which implies regular shops have also positive prices. There are i number of constraints for the θ variable, so choose i, where δ_i is minimal

$$p_0^* = \frac{\tau\sum_{i=1}^{n}\delta_i}{6n} - \frac{\theta}{3} > \frac{\tau}{4}\left(\frac{\sum_{i=1}^{n}\delta_i}{n} - \min_i \delta_i\right). \tag{17}$$

Because the average of a set of numbers is not lesser than its minimum value, then the second term is positive or zero, therefore p_0^* is positive.

3.2. Location

In every market or city regular shops own a separated q^*_{Ai} long area, where they supply the demand. The only constraint is that there must be on their left and right side the webshop. Their exact location can not be determined, only intervals can be given. This is ensured by the existence of Nash equilibrium according to Economides [1984], ie. the location setting in this model is the result of the behavior of companies. The theorem means that every regular shop must have enough long demand on both side in such a way, that they are not influenced in their profit maximizing by the other regular shop and the endpoints of the city. Therefore, the webshop locates on the fringes and between the two shops, moreover its price with the fixed delivery cost is the reservation price, k, which prevents the regular shops to raise prices too high. As a result of that regular shops can behave as local monopolies and in this situation they have no incentive to relocate.

Lets calculate (1) and (2) in equilibrium, which demonstrates that the demand on the left and right side on a company equals, $q_{Ai}/2$, ie. total q_{Ai}

$$x_{0A_i} = \frac{1}{\tau}(\tau x_{A_i} + p_{A_i} - p_0 - \theta) = x_{A_i} - \frac{q_{A_i}}{2} = x_{A_i} - \frac{\theta}{3\tau} - \frac{\sum_{i=1}^{n}\delta_i}{12n}, \tag{18}$$

$$x_{A_i0} = \frac{1}{\tau}\left(p_0 + \theta - p_{A_i} + \tau x_{A_i}\right) = x_{A_i} + \frac{q_{A_i}}{2} = x_{A_i} + \frac{\theta}{3\tau} + \frac{\sum_{i=1}^{n}\delta_i}{12n}. \tag{19}$$

So rearranging (18), we can get the left side of the interval, where the company A_i can be located. These locations ensure for company A_i to have the necessary demand on the left side, assuming x_{0Ai})>0.

$$\underline{x}_{A_i} = x_{0A_i} + \frac{q_{A_i}}{2} = x_{0A_i} + \frac{\theta}{3\tau} + \frac{\sum_{i=1}^{n}\delta_i}{12n} > \frac{\theta}{3\tau} + \frac{\sum_{i=1}^{n}\delta_i}{12n}. \tag{20}$$

The company A_i is constrained on the right side by the company B_i, therefore A_i should be at least q_{Ai} /2 distance from the last consumer of company B_i to ensure the necessary demand for both company. But we know B_i has also q_{Ai} demand due to symmetricity, so the right-most place for company A_i is the left of δ_i-3/2 q_{Ai}. The right side of the possible interval, where the company A_i can be located

$$\overline{x}_{A_i} = x_{0B_i} - \frac{q_{A_i}}{2} = \delta_i - \frac{3}{2}q_{A_i} = \delta_i - \frac{\theta}{\tau} - \frac{\sum_{i=1}^{n}\delta_i}{4n}. \tag{21}$$

The other company's location is analogous to company A_i's. The summarized results are the following

$$x_{A_i} \in \left(\frac{q^*_{A_i}}{2}, \delta_i - \frac{3q^*_{A_i}}{2}\right) = \left(\frac{\theta}{3\tau} + \frac{\sum_{i=1}^{n}\delta_i}{12n}, \delta_i - \frac{\theta}{\tau} - \frac{\sum_{i=1}^{n}\delta_i}{4n}\right), \tag{22}$$

$$x_{B_i} \in \left(\frac{3q^*_{A_i}}{2}, \delta_i - \frac{q^*_{A_i}}{2}\right) = \left(\frac{\theta}{\tau} + \frac{\sum_{i=1}^{n}\delta_i}{4n}, \delta_i - \frac{\theta}{3\tau} - \frac{\sum_{i=1}^{n}\delta_i}{12n}\right). \tag{23}$$

4. Comparing results

The results show similarities, especially in special case when $a,b = 0$ and $\delta_i = 1$ for all i: then constraints, prices and locations are identical. The constraint has a simple form, $2\theta < \tau$, ensuring the fixed delivery cost not to be too high, so webshop can enter to the markets. This results positive prices and the first company locates somewhere in the (1/12, 3/4) range, meanwhile the second company in the (1/4, 11/12) range in all markets. So the shops ensure enough space to each other to the optimal demand.

Of course, investigating the results in the general case show much more complicated situation. Table 1 helps to compare the two models. In the original model the variable a expresses the size of the external market. If we increase the value of a variable, then the prices raise. But we have to face an upper bound on market size and prices too due to the constraint. In the modified model it works slightly different, to the increase of the overall market or demand for webshop, the δi variables must be increased. Essentially this increases prices as they depend on the average of market sizes. If we suppose a fixed size for a market, for example that $\delta_1 = 1$ as in the original model, then we have to face also an upper bound due to the constraint.

Table 1: Main results of models

	Original model	Modified model
constraint	$\theta < \frac{\tau}{2}\left(1 - \frac{a}{b\tau + 2}\right)$	$\theta < \frac{3\tau}{4}\left(\min_i \delta_i - \frac{\sum_{i=1}^n \delta_i}{3n}\right)$
price of webshop	$\frac{\tau + \tau a - 2\theta}{2b\tau + 6}$	$\frac{\tau \sum_{i=1}^n \delta_i}{6n} - \frac{\theta}{3}$
price of regular shop	$\frac{\tau + \tau a + 4\theta + 2b\tau\theta}{4b\tau + 12}$	$\frac{\tau \sum_{i=1}^n \delta_i}{12n} + \frac{\theta}{3}$
location (left side of A)	$\frac{1}{\tau}\frac{\tau + \tau a + 4\theta + 2b\tau\theta}{4b\tau + 12}$	$\frac{1}{\tau}\left(\frac{\tau \sum_{i=1}^n \delta_i}{12n} + \frac{\theta}{3}\right)$
location (right side of A)	$\frac{1}{\tau}\frac{4b\tau^2 + 9\tau - 3\tau a - 12\theta - 6b\tau\theta}{4b\tau + 12}$	$\delta_i - \left(\frac{\sum_{i=1}^n \delta_i}{4n} + \frac{\theta}{\tau}\right)$

Table 2: Rewritten variables ($\delta_1 = 1$)

	Original model	Modified model
price of webshop	$\frac{\tau}{6\left(\frac{b\tau}{3}+1\right)} + \frac{\tau a}{6\left(\frac{b\tau}{3}+1\right)} - \frac{\theta}{3\left(\frac{b\tau}{3}+1\right)}$	$\frac{\tau}{6n} + \frac{\tau \sum_{i=2}^n \delta_i}{6n} - \frac{\theta}{3}$
price of regular shop	$\frac{\tau}{12\left(\frac{b\tau}{3}+1\right)} + \frac{\tau a}{12\left(\frac{b\tau}{3}+1\right)} + \frac{4\theta + 2b\tau\theta}{12\left(\frac{b\tau}{3}+1\right)}$	$\frac{\tau}{12n} + \frac{\tau \sum_{i=2}^n \delta_i}{12n} + \frac{\theta}{3}$
location (ls. of A)	$\frac{1}{\tau}\left(\frac{\tau}{12\left(\frac{b\tau}{3}+1\right)} + \frac{\tau a}{12\left(\frac{b\tau}{3}+1\right)} + \frac{4\theta + 2b\tau\theta}{12\left(\frac{b\tau}{3}+1\right)}\right)$	$\frac{1}{\tau}\left(\frac{\tau}{12n} + \frac{\tau \sum_{i=2}^n \delta_i}{12n} + \frac{\theta}{3}\right)$
		$1 - \frac{\tau}{4n} - \frac{\tau \sum_{i=2}^n \delta_i}{4n} - \frac{3\theta}{\tau 3}$

In the next *part* I use the assumption of the one unit long market and I rewrite the results (table 2). For simplicity, I suppose that the variables, which express the external market size, equal, $a = \sum_{i=2}^n \delta_i$ and the price sensitivity variables,

$\frac{b\tau}{3} + 1 = n$ or $b = \frac{3(n-1)}{\tau}$ $(b = 0 \Leftrightarrow n = 1)$.

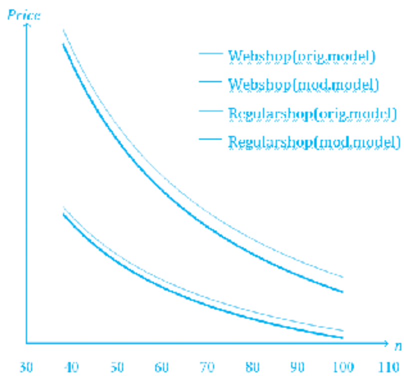
Looking at the last term of the webshop price and location in the original model:

$$\frac{4\theta + 2b\tau\theta}{12\left(\frac{b\tau}{3}+1\right)} > \frac{\theta}{3}. \tag{24}$$

It is clear that in this case the regular shop prices of the new model are lower and simple calculations show this is true also in the case of webshop prices. These effects are illustrated by the figure 1, where the external market is fixed and the number of shops is the changing variable.

There left an important question. If the markets are big enough for more than two

Figure 1: Prices in the original and modified model with fixed external market (τ = 2, θ = 0.1, a = 100)



regular shops, why new ones do not enter to the markets. The first option is the introduction of a constraint, which ensures that maximum two regular shops can operates in a market. Following this way, it must be true for all *i*, that $\delta_i/(q^*_{Ai})<3$. It is equivalent with

$$\max_i \delta_i < \frac{2\theta}{\tau} + \frac{\sum_{i=1}^{n} \delta_i}{2n}. \tag{25}$$

Moreover, θ has an upper constraint, (14). So together the two constraints ensure that every market bear only two regular shops.

$$\max_i \delta_i < \frac{2\theta}{\tau} + \frac{\sum_{i=1}^{n} \delta_i}{2n} < \frac{3}{2}\min_i \delta_i. \tag{26}$$

The other option is that we can consider this model as a short run model, where the number of shops is exogenous. In this case, there must be a long run model too, where shops enter to markets endogenously or maybe population can change by time. The goal of the article is not to demonstrate this model, because the complexity goes far beyond the framework of the model. The optimal number of regular shops would not be a continuous variable, so computational methods are required.

However, the results will be similar like in the previous case, if the long run model mean the endogenous number of the shops. The webshop always faces the residual demand, ie. the market size minus regular shops' demand, which can not be increased arbitrarily in the long run model supposing fixed number of markets. If one of the market grows and big enough, then a new shop may enter, which affects the residual demand of the webshop. Let's look shortly at the extension of the model, where we know the optimal regular shop sizes, ni for all markets. The numerator changes, but supposing ni = 2 for all i then we get back the previous results.

$$p_{L0}^{*} = \frac{\tau \sum_{i=1}^{n} \delta_i}{3 \sum_{i=1}^{n} n_i} - \frac{\theta}{3}, \tag{27}$$

$$p_{LA_i}^{*} = \frac{\tau \sum_{i=1}^{n} \delta_i}{6 \sum_{i=1}^{n} n_i} + \frac{\theta}{3}. \tag{28}$$

According to (27) and (28), prices of webshop and regular shops decrease with the raising the number of regular shops. So, compared to the short run results more shops may enter, which increases the competition. Moreover, this competition leads to smaller demand for regular shops according to (13), because the enter to the market is easier. On the other side this implies that the total demand of regular shops grows, so the residual demand of webshop constantly shrinks. Consequently, these results are much more competitive as before thanks to the higher number of shops.

5. Conclusion

The article showed a modification of Lijesen's webshop model [2013], who assumes the external demand as a linear function. The new model replaces that with n-1 different sized markets to try to give a more realistic framework. The results of the two models are similar, but there are some minor differences.

The formulas of prices, range of possible locations and constraints are almost identical using suitable rearrangements. The expand of external markets or the decrease the number of competitive regular shops increase prices in both case. But the structure of the new model is much more competitive, prices are lower and the possible locations of regular shops are wider.

The model can be regarded as a short run model, so the investigation of a long run version could be the next step of research. Of course, a new model, where population could increase in markets or could deal with new, entering shops, would be much more complex, as the profit function loses its continuity. Generally, it seems that endogenous number of regular shops would enhance the competition resulting lower prices.

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